# SHAPE OF DROPS AT THEIR FORMATION

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Shape characteristics of drops were measured experimentally in three phases of their formation. These characteristics measured at the moment of separation were compared with the existing theoretical relation. The shape characteristics of a drop before its separation is in linear dependence on the shape characteristics just after separation.

The process of drop formation was studied by many authors especially in relation to determination of interfacial or surface tension by the method of weighing the drops[1-4]. In many studies on drop formation an effort has been made to determine the volume of the formed drop in dependence on physical properties of the used system, wetted perimeter of the hole from which the drop forms and the rate of flow of the dispersed phase into the nozzle[5]. In operations in which during the drop formation transfer of mass, energy or electric charge takes place across the interfacial area of the drop, is also of importance the magnitude of interfacial area and thus the shape and shape changes of drops[6-8].

Theoretical description of shape changes of drops in the period of their formation could not have been derived as yet with respect to the formal mathematical difficulties at the solution of the Navier–Stokes equation together with the capilarity equation[9,10]. Moreover, neither the experimental studies on drop formation have led to development of certain rules[3,4]. Even in quasistatic formation the shape changes of drops at the moment of their separation are quite complex. Detachment of a drop proceeds in different manners even in systems with closely similar physical properties[3,4]. Here an effort has been made to find relations of the shape of drops at the beginning and at the end of the process of separation.

## THEORETICAL

During formation of a drop its shape is determined by the resultant of gravity, capilary and flow forces. In the case of the so-called quasistationary formation when the effect of flow forces can be neglected, the shape of a drop is given at each moment by the solution of the equation[1,2,9,10]

$$\bar{y}'' = 2\left(\frac{1}{\bar{h}} - \bar{y}\right)[1 + (\bar{y}')^2]^{3/2} - \frac{\bar{y}'}{\bar{x}}[1 + (\bar{y}')^2], \quad (1)$$

$$\bar{y}(\bar{x} = 0) = 0 ; \quad \bar{y}'(\bar{x} = 0) = 0 ,$$

where $x$ and $y$ are denoting the horizontal and vertical coordinate of the point on the drop profile (the origin of coordinates is assumed at the apex of the drop, the coordinate axis $y$ is identical with the axis of symmetry of the droplet) $h$ the curvature of the drop apex. The dash above the symbol denotes that the corresponding quantity is dimensionless due to division by a capilary constant $a = (2\sigma/\Delta\varrho \boldsymbol{g})^{1/2}$, where $\sigma$ is the interfacial or surface tension, $\Delta\varrho$ is the difference of densities of both phases, $\boldsymbol{g}$ gravity acceleration, $y''$ and $y'$ are the first and second derivatives of $y$ according to $x$.

Let us define the characteristics of the shape $E$ of a separated drop as the ratio of the maximum vertical and horizontal drop dimension; in the case of the still pendant drop instead of the vertical dimension the vertical distance of the narrowest point of the neck of the drop from its apex is considered. For the maximally still pendant drop[1,2,10] it is possible to derive from the numerical solution of Eq. (*1*) the dependence $E = E(\bar{r})$. For $\bar{r} < 0{\cdot}6$ the shape of this function can be[6] approximated with a maximum deviation of 1% by the four-term

$$E = 0{\cdot}99031 - 3{\cdot}4537\,\bar{r} - 1{\cdot}9241\,\bar{r}^4 + 3{\cdot}3000\,\bar{r}^{2/3}, \quad (2)$$

where $r$ is the radius of the nozzle.

## EXPERIMENTAL

Drops were formed by the liquid flow from nozzles submerged into the still bed of continuous phase and formation of drops was filmed at the velocity of 100 shots per s. From the shots the linear dimensions of drop patterns were measured with an accuracy corresponding to the maximum error of 3%. The characteristics $E$ were evaluated in three various moments: *1*) Before the beginning of separation which was estimated from changes in the dimension of the drop neck; the moment of separation is determined as the moment in which the contraction of the neck is accelerated; *2*) immediatelly after separation of the drop from the column of the prolonged neck; *3*) from the moment when the drop after its separation has the most prolate shape (this takes place in the time given approximately by the relation according to Lamb[7] or Miller and Scriven[8] for time of an oscillation period of infinitelly small oscillations of a stationary liquid sphere). The time interval between the beginning and end of the separation was in all low-viscous systems comparable with the corresponding time of a period of droplet oscillations. A survey of the systems used and of experimental conditions is given in Table I.

Originally the used liquids were analytical grade reagents and were further purified by a repeated fractional distillation on glass packed columns. The presented results are mean values from three shots of the drop in the same situation. The difference between these individual measurements was always smaller than the maximum experimental error.

All the experiments were performed with mutually saturated liquids at 25°C.

## RESULTS AND DISCUSSION

Experimental characteristics $E$ were compared to the theoretical dependence of the function $(r)$ given in Fig. 1. Empty points represent characteristics $E$ in the moment of the assumed beginning of the drop separation, halved-points are related to the shape of the just separated drop. Full points characterize the maximum prolate shapes of drops after the period of shape changes since the separation. Numbers in the upper part of the Fig. represent the used system (Table I). Curve *I* corresponds to the theoretical shape of function $E(\bar{r})$ according to Eq. (2). Though the phenomenon of drop separation is very complex, the shape before and after their separation are in a very close relation. Curve *II*, around which are situated points concerning the shape of drops at the end separation, is in a simple relation to the curve *I*:

$$E_{\mathrm{I}} - 1\cdot0 = 3\cdot5\,(E_{\mathrm{II}} - 1\cdot0)\,. \tag{3}$$

The agreement of the shape of curve *I* with the points characterizing drops at the beginning of separation is not surprising as at small and moderate velocities of drop

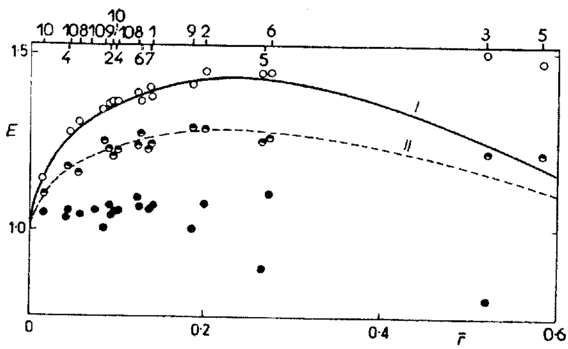


FIG. 1

Characteristics of Droplet Shapes

Curve *I* theoretical shape of the characteristics $E(\bar{r})$ according to Eq. (2), curve *II* curve affine to curve *I*; ○ points related to the beginning of the drop separation, ◒ points related to the end of drop separation, ● points characterizing the shape of maximum prolate of the drop after separation. Numbers in the upper part of the Fig. represent the system of compounds according to Table I.

formation its shape is only little affected by the inflowing liquid[9,10]; moreover, under the used experimental conditions the actual volume of the formed drop is quite close to the quasistationary drop volume so that even the eventual error in determination of the moment of separation does not lead to significant deviations of the determined shape characteristics from their theoretical values of the maximum still stable pendant drop.

The moment at which the separation is terminated has been determined with the maximum error, of 0·01 s. Closely after separation of the drop, vigorous and considerable changes in the shape of drops take place so that the value of the characteristics $E$ is considerably dependent on the selected time of the end of separation. For comparison in Fig. 1 only shapes of drops found on shots were included in which the shapes of drops were taken only a few thousand parts of the second after the end of their separation. So determined characteristics represent the upper limit of all values found in the whole interval from 0 to 0·01 s after termination of separation.

After the separation an accelerated motion of drops took place while the shape changes taking place can be in general characterized as dumped periodical oscillations between the prolate and oblate shape. For completeness in Fig. 1 is also plotted the shape characteristics in the moment of maximum drop prolongation after its separation. These shapes are already not in some simple relation to the shape of the curve *I*.

The points which deviate for $\bar{r} > 0·5$ were found in systems with extreme physical properties (high viscosity of butanol and very low interfacial tension in the system

TABLE I
Used Systems

| System | Dispersed phase | Continuous phase | Capillary constant | Flow of dispersed phase through the nozzle |
|---|---|---|---|---|
| | | | | $10^{-9}$ $m^3/s$ |
| 1 | water | n-hexane | 0·546 | 19·8 |
| 2 | $CCl_4$ | water | 0·382 | 1·7 |
| 3 | water | n-butanol | 0·147 | 1·7 |
| 4 | aniline | water | 0·789 | 0·7 |
| 5 | aniline | n-heptane | 0·131 | 0·7 |
| 6 | water | n-heptane | 0·277 | 2·2 |
| 7 | water | diethyl ether | 0·550 | 2·2 |
| 8 | water | cyclohexane | 0·615 | 6·6 |
| 9 | water | cyclohexanol | 0·411 | 2·2 |
| 10 | water | benzene | 0·762 | 6·6 |

aniline–n-heptane). Also in these systems are the moments of beginning and end of separation especially indefinite. Therefore, the deviation of the mentioned points cannot be considered as a result of some systematic deviation but a consequence of a specific experimental error.

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